MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
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"National Research Lobachevsky State
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PROGRAM OF ADMISSION TESTS IN MATHEMATICS

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During the admission test in mathematics, university entrants should demonstrate:

- a clear knowledge of mathematical definitions and theorems envisaged by the program, the ability to apply them with sufficient justification for solving problems;
- the ability to express a mathematical idea in a precise and concise manner in oral and written form, to use the appropriate symbolism;
- confident mathematical knowledge and skills envisaged by the program, the ability to use them when solving problems.

This mathematics program for university entrants consists of three sections. The first section lists the basic mathematical concepts that the entrant must know and must demonstrate this knowledge in the written and in the oral exam. It also contains the basic formulas and theorems that the entrant must be able to prove.

The second section is a list of questions for the theoretical part of the oral exam. The third section describes the skills and abilities the applicant is required to demonstrate at the test that will be conducted in oral and written form.

The amount of knowledge and degree of knowledge of the material described in the program correspond to the course of high school mathematics. An applicant can use the entire arsenal of tools from this course, including pre-calculus. However, for solving problems, it is sufficient to confidently know only those concepts and properties that are listed in this program. Objects and facts that are not covered by the comprehensive school curriculum can also be used by applicants, but on condition that they are able to explain and prove them.

There are many different mathematics textbooks in different countries, however, university entrants should know the basic formulas and theorems listed in Section 2 of this program regardless of possible variations in their names and formulations.

1. BASIC MATHEMATICAL NOTIONS AND FACTS

Arithmetic, algebra and pre-calculus

1. Natural numbers (N). Divisibility. Simple and composite numbers. Divisor, multiple. The greatest common divisor. The lowest common multiple.
2. Criteria of divisibility by 2, 3, 5, 9, 10.
3. Integers (Z). Rational numbers (Q), their addition, subtraction, multiplication and division. Comparison of rational numbers.
4. Real numbers (R), their representation in the form of decimal fractions. Rational and irrational numbers. Percent. Comparison of real numbers.

5. Showing numbers on the line. The modulus of a real number, its geometric meaning.
6. Numeric expressions. Expressions with variables. Formulas of abridged multiplication. Equalities and identities.
7. Degree with a natural and rational exponent. Arithmetic root. Properties of arithmetic roots of degree n. Degree with a real exponent.
8. Monomial and polynomial. Polynomial of one variable. Algebra of polynomials.
9. The concept of function. Ways of setting a function. Function domain, set of function values.
10. Function plot. Increasing and decreasing functions, periodicity, evenness and oddness. The largest and smallest value of the function.
11. Definition of a derivative. Its physical and geometric meaning. The derivative of the sum, product and quotient of two functions. Equation of a tangent to the function curve. Derivatives of the functions $y=\sin x$; $y=\cos x$; $y=\operatorname{tg} x$; $y=\operatorname{ctg} x$; $y=x^{\alpha}(\alpha \in R)$; $y=a^{x}$; $y=\ln x$.
12. Sufficient condition for an increasing (decreasing) function on an interval. The concept of a function extremum. The necessary condition for the extremum of a function (Fermat's theorem). Sufficient condition of an extremum. The function's largest and smallest value on an interval.
13. Definition, basic properties and graphs of functions: linear $y=ax+b$, quadratic $y=ax^2+bx+c$, power $y=ax^b$, hyperbola $y=a/x$, exponential $y=a^x (a>0)$, logarithmic $y=\log_a x$, trigonometric functions ($y=\sin x$, $y=\cos x$, $y=\operatorname{tg} x$, $y=\operatorname{ctg} x$), arithmetic root $y=\sqrt[n]{x}$. Graph of the function $y=a|x|$.
14. Equation. Roots of an equation. The concept of equivalent equations. Solving linear and quadratic equations and those reduced to quadratic equations. Formula of the roots of a quadratic equation. Decomposition of a quadratic trinomial into linear factors. Viete's theorem.
15. Inequalities. Properties of inequalities. Proof of inequalities. The concept of equivalent inequalities. Intervals method.
16. Systems of equations and inequalities. Equivalence of systems. The solution of systems of equations and systems of inequalities.
17. Arithmetic and geometric progression. The formula of the *n*-th member and the sum of the first members of the arithmetic progression. The formula of the *n*-th member and the sum of the first *n* members of a geometric progression.
18. Logarithms. Properties of logarithms. Equations and inequalities containing exponential and logarithmic functions.
19. Trigonometric functions. The relationship between the functions of the same argument. Reduction formulas. Sine and cosine of the sum (difference) of two arguments. Trigonometric functions of double and half arguments. Conversion of the sums of $\sin x \pm \sin y$, $\cos x \pm \cos y$ to the product. Conversion of the product of $\cos x \cos y$, $\sin x \sin y$, $\cos x \sin y$ to sums.

20. The solution of the equations $\sin x = a$, $\cos x = a$, $\operatorname{tg} x = a$, $\operatorname{ctg} x = a$ and the equations to which they are reduced. The simplest trigonometric inequalities.

Geometry

1. Straight line, ray, segment, polyline; length of the segment. Angle, angle value. Vertical and adjacent angles. Circle, solid circle. Parallel straight lines. Criteria for parallel lines. Properties of points equidistant from the ends of the segment.

2. Examples of transformation of figures, types of symmetry. Conversion of similarity and its properties.

3. Vectors. Vector operations.

4. Polygon. Its vertices, sides, diagonals.

5. Triangle. Its medians, bisectors, altitudes. Their properties. Types of triangles. Relations between the sides and corners of a right triangle. Properties of an isosceles triangle. The sum of the corners of the triangle. Criteria for similarity of triangles. Pythagorean theorem.

6. Quadrangle: parallelogram, rectangle, rhombus, square, trapezium. The sum of the inner corners of a convex polygon. Criteria for a parallelogram.

7. Circumference and circle. Center, chord, diameter, radius. Tangent line to the circle. Arc of a circle. Sector. Central and inscribed angles, their measurement. Tangent line and secant to the circle, their properties. Properties of chords of circles. The angle between the chords. The angle between the tangent and the chord drawn through the tangency point.

8. Circles circumscribed around the triangle and inscribed in it. The existence of circles circumscribed around a quadrilateral or inscribed in it.

9. The formula of the area for: a triangle, a rectangle, a parallelogram, a rhombus, a square, a trapezoid.

10. The length of the circle and the length of the arc of a circle. Degree and radian measure of angles and arcs of circles. Circle area and sector area. The formula for the distance between two points of the plane. The equation of a circle.

11. Similarity. Similar figures. The ratio of the areas of similar figures.

12. Plane. Parallel and intersecting planes. Criteria of parallel planes.

13. Parallelism of a line and a plane. The criterion of parallelism of a line and a plane.

14. The angle between a straight line with a plane. Perpendicular to the plane. The perpendicularity theorem for a line and a plane.

15. Dihedral angles. Linear angle of the dihedral angle. Perpendicularity of two planes. The theorem on the perpendicularity of two planes.

16. Polyhedra. Their vertices, edges, faces, diagonals. Straight and oblique prisms; pyramids. The regular prism and the regular pyramid. Parallelepipeds, their types.

17. Figures of rotation: cylinder, cone, sphere, solid sphere. Center, diameter, radius of a sphere and a solid sphere. The plane tangent to the sphere.
18. The formula for the surface area and volume of the parallelepiped.
19. The formula for the surface area and volume of the prism.
20. The formula for the surface area and volume of the pyramid.
21. The formula for the surface area and volume of the cylinder.
22. The formula for the surface area and volume of the cone.
23. The formula for the surface area of the sphere.
24. The formula for the volume of the ball.

2. BASIC FORMULAS AND THEOREMS

Algebra and pre-calculus

1. Properties of the function $y=ax+b$ and its graph.
2. Properties of the function $y=a/x$ and its graph.
3. Properties of the function $y=ax^2+bx+c$ and its graph.
4. The formula of the roots of a quadratic equation.
5. Formulas of abridged multiplication.
6. The decomposition of the quadratic trinomial into linear factors.
7. Properties of numerical inequalities.
8. The logarithm of the product, degree, quotient.
9. The definition and properties of functions $y=\sin x$ and $y=\cos x$ and their graphs.
10. The definition and properties of functions $y=\operatorname{tg} x$ and $y=\operatorname{ctg} x$ and their graphs.
11. Solution of equations of the form $\sin x=a$, $\cos x=a$, $\operatorname{tg} x=a$, $\operatorname{ctg} x=a$.
12. Reduction formulas.
13. Dependence between trigonometric functions of the same argument.
14. Trigonometric functions of double argument.
15. The derivative of the sum of two functions.
16. The equation of the tangent line to the graph of the function.

Geometry

1. Properties of an isosceles triangle.
2. Properties of points equidistant from the ends of a segment.
3. Criteria for parallel lines.
4. The sum of the angles of a triangle. The sum of the inner corners of a convex polygon.

5. Criteria for a parallelogram, its properties.
6. A circumference circumscribed around a triangle.
7. A circle inscribed in a triangle.
8. Tangents to the circle and their property.
9. Measurement of the angle inscribed in a circle.
10. Criteria for similarity of triangles.
11. Pythagorean theorem.
12. Formulas of areas of parallelogram, triangle, trapezium.
13. The formula of the distance between two points of the plane.
14. Equation of a circle.
15. The criterion for parallelism of a straight line and a plane.
16. The criterion for parallel planes.
17. The theorem on the perpendicularity of a straight line and a plane.
18. Perpendicularity of two planes.
19. The theorem on parallelism and perpendicularity of two planes.
20. The theorem of three perpendiculars.

4. REQUIREMENTS FOR THE APPLICANT

A the test in mathematics, the applicant should be able to:

1. Perform (without a calculator) operations on numbers specified in the form of decimal or ordinary fractions, round these numbers and the results of calculations with the required accuracy; perform operations on numeric expressions; if necessary, one can use calculators to make auxiliary calculations, but not as a way of proof; convert letter expressions; convert units of measurement of values to other units.
2. Compare numbers (without a calculator); carry out identical transformations of polynomials, fractions containing variables, expressions containing power, exponential, logarithmic and trigonometric functions; prove identities and inequalities for letter expressions.
3. Solve equations and inequalities of the first and second degree, as well as the equations and inequalities reduced to them, solve systems of equations and inequalities of the first and second degree reduced to them (including those with parameters) and research their solutions. This includes, in particular, certain algebraic equations and inequalities of higher degrees, irrational equations and inequalities, simple equations and inequalities containing power, exponential, logarithmic and trigonometric functions.
4. Investigate functions; graph functions and sets of points on the coordinate plane specified by equations and inequalities. Use the notion of derivative in the

study of functions for increasing (decreasing), for extremum, and for graphing functions.
5. Draw geometric figures in a drawing; make additional constructions; construct sections; explore the relative position of the figures; apply criteria of equality, similarity of figures and their belonging to a particular type.
6. Use the properties of numbers, vectors, functions and their graphs, the properties of arithmetic and geometric progressions.
7. Use the properties of geometric figures, their characteristic points, lines and parts, the properties of equality, similarity and relative position of the figures.
8. Use ratios and formulas containing modules, degrees, roots, logarithmic, trigonometric expressions, angles, lengths, areas, volumes.
9. To make equations, systems, inequalities, based on the conditions of the problem; solve problems that involve making equations and find the values of quantities based on the statement of problem.
10. Perform operations on vectors on the plane (addition and subtraction of vectors, multiplication of a vector by a number, scalar product), use the properties of these operations.
11. To present the solution in a logically correct, full and consistent way, with the necessary explanations.

ASSESSMENT CRITERIA
FOR GRADING AT THE ADMISSION EXAM
in mathematics

Each ticket consists of 5 practical tasks. The solution of the selected tasks is assessed using a 100-point scale (each task is assessed at 20 points).

The final score is formed by summing up the points for individual tasks. The solution of the task must be accurate, complete and exhaustive. Mathematical terminology must be used correctly. All transformations must be mathematically strict and reasonable. If the formulation of the conditions of the problem allows for several options, then all options should be considered in detail and a solution should be given for each option.

Scoring Table

№	Criteria for the answers	Points
1	Complete, correct, comprehensive solution of the problem	20
2	The solution is correct, but there are some minor computational and/or logical errors in the solution.	15
3	The solution as a whole is correct, but it contains gross computational and/or logical errors, or the problem has not been completely solved, however, the main case has been considered	10
4	There is an initial progress in solving the problem.	5
5	The solution is missing or is incorrect	0

Depending on the seriousness of the mistakes made by the applicant, the score may be adjusted up or down in the range from –4 to +4 points.
Insignificant inaccuracies that did not result in a distortion of the meaning of the findings or proofs (for example, careless writing, careless design of graphs, drawings, etc.) do not affect the grade.